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Pressure drop correlations for two-phase flow within horizontal rectangular channels with small heights

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Abstract

The present paper proposes new correlations for the two-phase pressure drop through horizontal rectangular channels with small gaps (heights) based on 305 data points. The gap between the upper and the lower plates of each channel ranges from 0.4 to 4 mm while the channel width being fixed to 20 mm. Water and air were used as the test fluids. The superficial velocity ranges of water and air were 0.03–2.39 and 0.05–18.7 m/s, respectively. The atmospheric pressure condition was maintained throughout the experiments. In the present study, the two-phase frictional multiplier was expressed using the Lockhart–Martinelli type correlation but with the modification on parameter C. Effects of the mass flux and the gap size were considered. The correlations with the modified C successfully cover wide ranges of the Martinelli parameter ($X = 0.303$–79.4) and the all-liquid Reynolds number ($Re_{\mathrm{Lo}} = 175$–17700) based on the hydraulic diameter within the deviation of $\pm10\%$.

Keywords: Two-phase flow; Horizontal rectangular channel; Frictional pressure drop; Two-phase frictional multiplier; Lockhart–Martinelli type correlation

1. Introduction

Most of the researches reported on the two-phase flow behavior were with the circular tube larger than 10 mm in diameter. However, recently, the flow behavior within small tubes (including the narrow rectangular channels with their hydraulic diameters less than 5 mm) is becoming of interest. Such small tubes are widely adopted in compact heat exchangers because of their high heat transfer performance. Despite this advantage, there is also a demerit; the pressure drop

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through the tube increases with the smaller tube diameter due to the increase of the wall friction. Especially, inside the evaporators and condensers, the two-phase mixture flows with various flow patterns and qualities, and the pressure drop becomes even larger. From the viewpoint of the optimal design of heat exchangers, it is desirable to promote the heat transfer performance without increasing the pressure drop. Therefore, exact estimation of the pressure drop is essential in the design procedure of heat exchangers . Nevertheless, a limited number of works have been reported on the two-phase flow patterns and the frictional pressure drop with the small channels. The studies on the two-phase frictional pressure drop through the rectangular channels were performed by Lowry and Kawaji (1988), Ide and Matsumura (1990), Wambsganss et al. (1992), Mishima et al. (1993), Fujita et al. (1995) and Mishima and Hibiki (1996). The channel configurations of the previous works along with those of the present work are summarized in Table 1.

Lowry and Kawaji (1988) have examined the flow patterns of the cocurrent upward air/water flow in narrow passages with the gap size between 0.5 and 2 mm, and measured the pressure drop along the passage. They concluded that the Lockhart–Martinelli correlation is an adequate predictor of the two-phase frictional multiplier for the pressure drop, but fails to predict the mass velocity effect. Instead, they argued that the two-phase frictional multiplier is mainly dependent on the superficial gas velocity and less sensitive to the liquid velocity and the gap width.

Ide and Matsumura (1990) performed a series of experiments using 10 rectangular channels with different aspect ratios and inclination angles. They reported that the conventional Lockhart–Martinelli correlation could not represent the experimental results at the condition of low liquid superficial velocity and high inclination angle. For the channels with their hydraulic diameters ranging from 7.3 to 21.4 mm, they proposed a correlation to predict the frictional pressure drop in terms of the aspect ratio, inclination angle, liquid Reynolds number and the void fraction based on the separated flow model.

Wambsganss et al. (1992) measured frictional pressure gradient of air–water mixture in a rectangular channel with a cross-section of 19.05 mm $\times$ 3.18 mm but in different horizontal orientations (i.e., with the aspect ratios of 1/6 and 6). They proposed a correlation for parameter C as a function of the Martinelli parameter (X) and the all-liquid Reynolds number Re_{Lo} for limited ranges ($X < 1,\ Re_{\mathrm{Lo}} < 2200$).

On the other hand, Mishima et al. (1993) and Mishima and Hibiki (1996) reported that parameter C in Lockhart–Martinelli type correlation should include the effect of tube size to represent the frictional pressure drop for small tubes less than 5 mm in hydraulic diameter.

Table 1
Dimensions of the test sections

Data source	Flow direction	Gap × Width (mm)	Hydraulic diameter (mm)
Present work	Horizontal	$(0.4, 1, 2, 4) \times 20$	0.78, 1.91, 3.64, 6.67
Lowry and Kawaji (1988)	Vertical	$(0.5, 1, 2) \times 80$	0.99, 1.98, 3.9
Ide and Matsumura (1990)	Vertical	$(4–14.6) \times (14–160)$	7.3–21.4
	Horizontal to Vertical		
Wambsganss et al. (1992)	Horizontal ($\alpha > 1$)	3.18×19.08	5.45
	Horizontal ($\alpha < 1$)	19.08×3.18	
Mishima et al. (1993)	Vertical	$(1.07, 2.45, 5) \times 40$	2.08, 4.62, 8.89
Fujita et al. (1995)	Horizontal ($\alpha > 1$)	$(0.2 - 2) \times 10$	0.4–3.3

Fujita et al. (1995) also reported that, based on the experiments with the rectangular channels (gap size ranges between 0.2 and 2.0 mm), the Lockhart–Martinelli correlation cannot predict the two-phase pressure drop when the liquid superficial velocity is low and the flow pattern is in an intermittent regime.

All the above works can be summarized as follows: Firstly, the two-phase flow pressure drop cannot be represented properly by the conventional Lockhart–Martinelli correlation for the two-phase frictional multiplier and need modifications somehow. Secondly, the two-phase frictional multiplier should take account of the mass flow rate, and the gap size especially with the narrow gap. Therefore, in the present study, a series of experiments have been performed to find out the correlations representing the frictional pressure drop of the two-phase horizontal flow within the narrow-gap rectangular channels.

2. Experiments

Fig. 1 shows the experimental setup for the measurement of the two-phase pressure drop. The system basically consists of the water and the air supply lines with the flowmeters and the valves to measure and control the flow rates. The air flow rate was measured using the rotameter with three different measurable ranges, i.e., 0.5, 5 and 50 l/min. The air supply line is connected to the building air source through a filter and a regulator. The water flow rate was measured using a rotameter and a turbine type flowmeter with the maximum measurable ranges up to 0.7 and 25 l/min, respectively. In order to introduce the bubbles uniformly to the test section, a porous plate with 40-μm pore size was installed at the test section inlet. The cross-section of the test section has the rectangular shape with the height much smaller than the width. The upper plate of the test

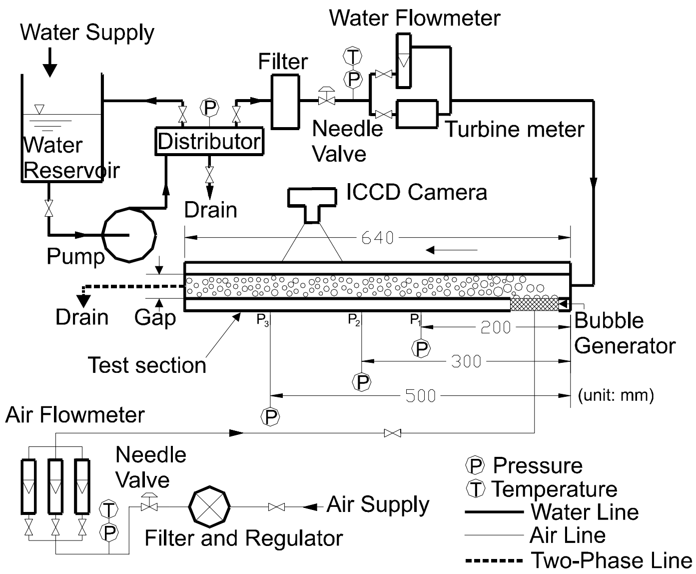

Fig. 1. Schematic diagram of the experimental setup.

section is made of transparent acrylic plates for flow visualization. Four test sections were prepared with diff erent gap sizes; the gap sizes are 0.4, 1, 2 and 4 mm while the width of the channel is fixed to 20 mm. The entire length of the test section is 640 mm, and three pressure taps (points P_1, P_2 and P_3) were drilled along the centerline of the lower plate of the test section. Point P_1 is to monitor the reference pressure of the test section and the pressure drop is measured between points 2 and 3 using the differential pressure gages with different measurable ranges (± 14, ± 35 and ± 55 kPa) corresponding to the experimental conditions. The signals from the pressure transducers are amplified and sampled with the sampling rate of 200 Hz for 20 s using the data acquisition system. In order to estimate the properties of the air and the water flowing into the test section, the temperatures of the air and water were measured at the inlets of the flowmeters using the thermocou ples along with the two-channel thermometer. The uncertainty analysis has been performed according to the method proposed by Kline (1985). The estimated uncertainties of the flow rates and the pressure measurements are $\pm 4\%$ and $\pm 2\%$, respectively.

3. Results and discussion

Prior to performing the two-phase flow experiments, the friction factor for the single-phase (air) flow was obtained to check the reliability of the experimental system. The friction factors for the laminar air flow through the rectangular channels were obtained by using the simplified (polynomial type) equation by Hartnett and Kostic (1989), that fits the exact analytical solution within an accuracy of 0.05%.

$$f\,Re_{D_h} = 24(1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5), \tag{1}$$

where f is the friction factor and D_h is the hydraulic diameter. Also, α denotes the aspect ratio defined as the value of the height divided by the width of the channel cross-section. For single-phase turbulent flows, the friction factor can be expressed by the Blasius equation as:

$$f = 0.079\,Re_{D_h}^{-0.25}. \tag{2}$$

As seen in Fig. 2, the experimental results agree with Eqs. (1) and (2) very precisely for the laminar and the turbulent flows, respectively. The same results can be found from the works by Wambsganss et al. (1992), Mishima et al. (1993) and Olsson and Sunden (1995). Transition from the laminar to the turbulent flows occurs at the Reynolds number of about 2000.

The experimental ranges for the air and the water flow rates in two-phase experiments are listed in Table 2. Fig. 3 shows a typical variation of the pressure gradients with the superficial velocities of the liquid and the gas flows for gap sizes 0.4 and 4 mm, respectively. As imagined, the pressure gradient increases with the increases in the superficial velocities of air and water, j_G and j_L, and with the decrease in the gap size. From the measured results, the two-phase frictional multiplier ϕ_L^2 was expressed as a function of the Martinelli parameter as shown in Fig. 4. The two-phase frictional multiplier and the Martinelli parameter are defined as:

$$\phi_L^2 = \left(\frac{dp}{dz}F\right)_{TP} \Big/ \left(\frac{dp}{dz}F\right)_L, \tag{3}$$

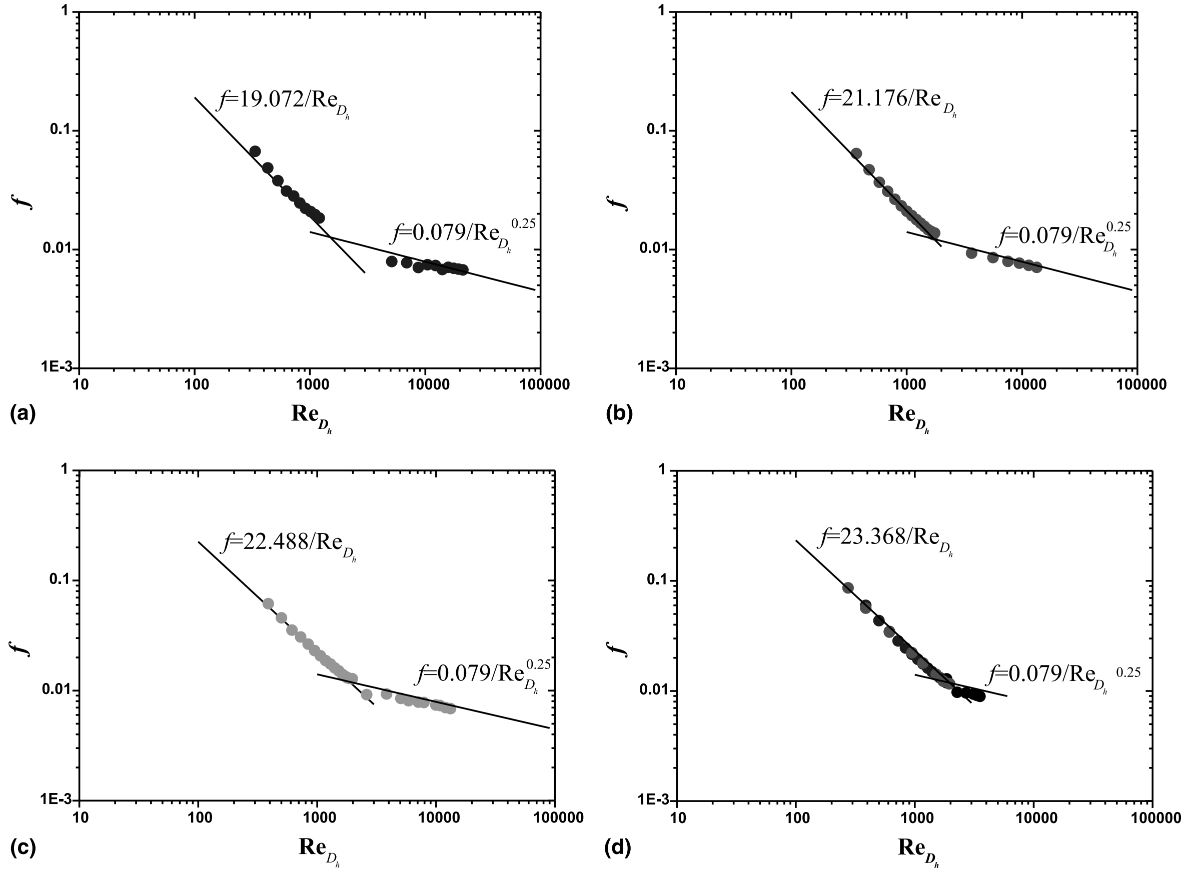

Fig. 2. Friction factors for single-phase flows in rectangular channels: (a) Gap 4 mm; (b) Gap 2 mm; (c) Gap 1 mm; (d) Gap 0.4 mm.

Table 2
Ranges of the superficial velocities of the liquid (water) and gas (air) tested

Gap (height), mm	0.4	1.0	2.0	4.0
j_L (m/s)	0.26–2.66	0.07–2.06	0.05–2.03	0.05–2.05
j_G (m/s)	0.17–18.7	0.1–16.3	0.05–14.8	0.03–7.39

$$X = \left[\left(\frac{dp}{dz} F \right)_L \Big/ \left(\frac{dp}{dz} F \right)_G \right]^{1/2}, \tag{4}$$

where $(dp/dz)F$ is the frictional pressure gradient and subscript TP denotes the two-phase mixture. It should be noted that, in the present adiabatic (air–water) flow case, no heat transfer is involved. Hence, the quality remains unchanged and the acceleration component of the two-phase pressure drop is neglected here. The two-phase frictional multiplier is obviously smaller with the smaller gap size as shown in Figs. 4(a)–(d).

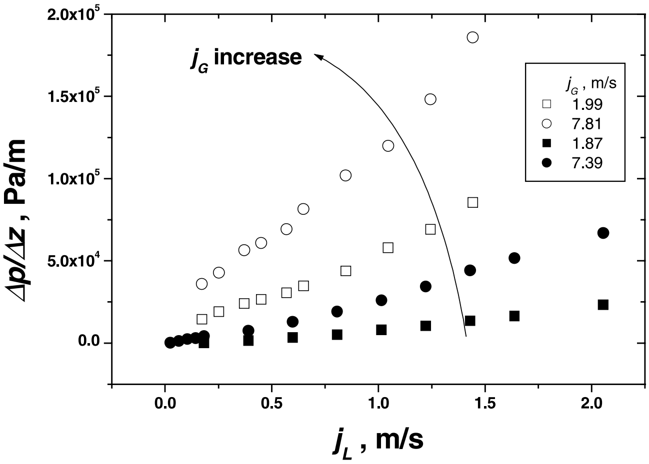

Fig. 3. Typical variation of the pressure gradient with the superficial velocities of the liquid and the gas flows (solid symbols: Gap 4 mm, open symbols: Gap 0.4 mm).

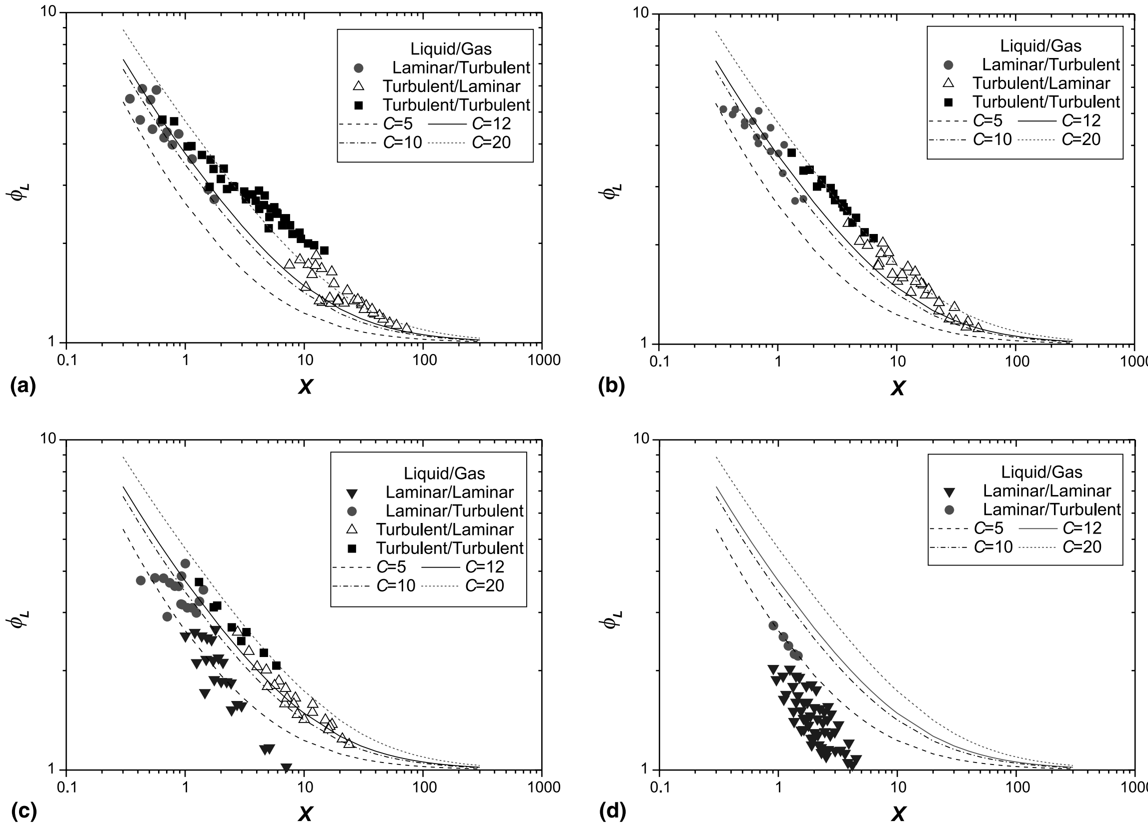

Fig. 4. Comparison of the present measurements with Lockhart–Martinelli correlation: (a) Gap 4 mm; (b) Gap 2 mm; (c) Gap 1 mm; (d) Gap 0.4 mm.

In order to represent the experimental results, the classical Lockhart–Martinelli type correlation may be used as follows:

$$\phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2}. \tag{5}$$

In the classical literature, the constant values have been proposed for parameter C for each flow regime as listed in Table 3 (Chishlom (1967)). In this Table, Re_L and Re_G stand for the Reynolds numbers with the liquid and the gas phases considered flowing alone in the channel, respectively. The value of C is large ($C = 20$) at the higher flow rate (i.e., in turbulent–turbulent regime) whereas small ($C = 5$) at the low flow rate (i.e., in laminar–laminar regime), and is in-between ($C = 12$ or 10) at the intermediate flow rate (i.e., in laminar–turbulent or turbulent–laminar regime). However, as seen in Fig. 4, Eq. (3) with the constant values for C cannot represent the experimental data well, especially the cases with the small gap size. Thus, if the Lockhart–Martinelli type correlation is to be used, parameter C should be expressed not only with the flow rate regimes, but also with the gap size by any means.

Concerned with the effect of the gap size, Mishima and Hibiki (1996) has proposed a correlation for parameter C as a function of the hydraulic diameter as follows:

$$C = 21\{1 - \exp(-0.319 D_h)\}. \tag{6}$$

In Fig. 5, the present experimental results are compared with Eq. (6). In the same figure, the results by Lowry and Kawaji (1988), Wambsganss et al. (1992), Mishima et al. (1993) and Fujita et al. (1995) are also shown. From the figure, Eq. (6) seems to represent the experimental results reasonably. However, the points in Fig. 5 are the averaged values for each gap size, and large deviations of the measured data from the averaged values are observed. The measured values of ϕ_L were compared with the prediction by Mishima and Hibiki (1996) (Eqs. (5) and (6)) as in Fig. 6, where the large deviations are still obvious. This is because Eq. (6) does not count the effect of the flow rates of the gas and the liquid, which are the important factor not to be excluded. This necessitates development of a new correlation that takes account of both the effects of the flow rates and the gap size.

At this stage, it is meaningful to review the physics meant by Eq. (5). The original concept of Eq. (5) came from the following equation:

$$\left(-\frac{dp}{dz}F\right)_{TP} = \left(-\frac{dp}{dz}F\right)_L + C\left[\left(-\frac{dp}{dz}F\right)_L \left(-\frac{dp}{dz}F\right)_G\right]^{1/2} + \left(-\frac{dp}{dz}F\right)_G. \tag{7}$$

Table 3
Parameter C in Lockhart–Martinelli correlation[a]

Liquid phase	Gas phase	C
Turbulent	Turbulent	20
Laminar	Turbulent	12
Turbulent	Laminar	10
Laminar	Laminar	5

[a] Laminar: $Re_L, Re_G < 1000$; turbulent: $Re_L, Re_G > 2000$ (Chishlom, 1967).

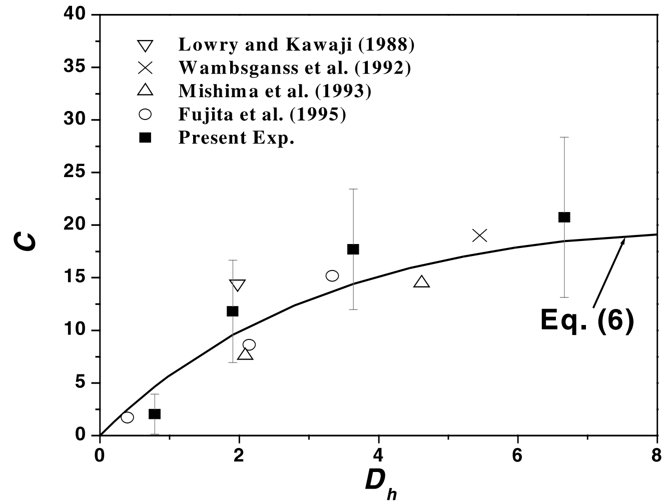

Fig. 5. Effect of the gap size (hydraulic diameter) on parameter C .

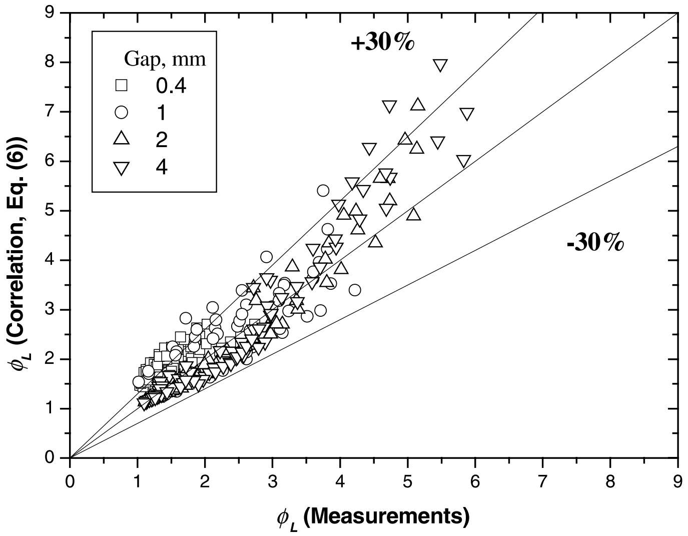

Fig. 6. Comparison between the present measurements and the correlation by Mishima and Hibiki (1996).

The two-phase pressure drop in the left-hand side of the above equation should be equal to the sum of the liquid-only and the gas-only pressure drops and the phase interaction in the right-hand side. Here, parameter C in Eq. (7) is a measure of the interaction between the phases, which depends on the flow regime; and as discussed earlier, the flow regime, in turn, depends on the flow rates and the gap size. In the present study, therefore, the flow was classified into four different regimes as was done by Lockhart and Martinelli. Then, the separate correlations for parameter C were proposed with both the effects of the flow rate and the gap size taken into account.

When Fig. 4 is carefully examined, the deviation of the data points for the case of 0.4 mm from the line representing $C = 5$ is found to be the largest. (If the gap size becomes smaller, the flow regime tends to be mostly laminar–laminar.) According to Lee and Lee (1999), the flow pattern in the laminar–laminar flow regime is mostly the plug or the slug flow as shown in the sketch of Fig. 7. In this case, the two-phase pressure drop can be divided into three components obviously. They are the liquid-phase and the gas-phase pressure drops, and the pressure drop by the phase interaction (mostly by the interface curvature). These pressure-drop components correspond to each term in the right-hand side of Eq. (7). When the gap size becomes smaller, the interface curvature decreases and the surface tension force become predominant. Fukano and Kariyasaki (1993) also presented in their paper that there is a critical tube diameter below which the surface tension effect becomes important; and the critical diameter exists between 5 and 9 mm.

Suo and Griffith (1964) have studied the surface-tension-force dominant flows through the horizontal capillary tubes with their radii being 0.5 and 0.8 mm. If the viscosity and the inertia (density) of the gas phase is negligibly small, the factors influencing the bubble velocity in the horizontal slug flow can be obtained by using the π-theorem as follows:

$$\frac{\rho_L j D_h}{\mu_L}, \quad \frac{j\mu_L}{\sigma}, \quad \frac{\rho_L g D_h^2}{\sigma}, \tag{8}$$

where ρ_L, j, μ_L, σ and g denote the liquid density, liquid slug velocity, liquid viscosity, surface tension, and the gravitational constant, respectively. The first group is the Reynolds number for the liquid slug. The second group represents the relative importance of the viscous and the surface tension effects and denoted by ψ as:

$$\psi = \frac{\mu_L j}{\sigma}. \tag{9}$$

The first and the second group can be combined to get a parameter independent of the liquid slug velocity (j) and is constant for a fluid as

$$\lambda = \frac{\mu_L^2}{\rho_L \sigma D_h}. \tag{10}$$

Therefore, dimensionless parameters ψ and λ in Eqs. (9) and (10) are the effects of the surface and the viscosity forces and the velocity of the liquid slug, j. The final group is the ratio between the gravity and the surface tension forces. For the horizontal slug flow in small channels, the third group of Eq. (8) is negligible; otherwise, the bubbles would not exist as described by the slug flow

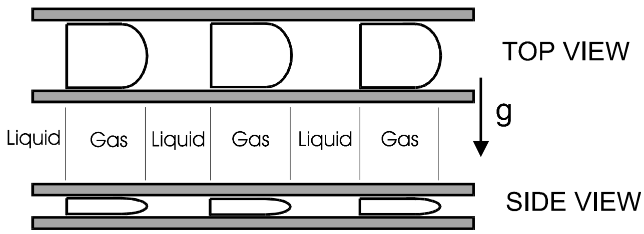

Fig. 7. Capillary slug flow in a horizontal rectangular channel.

regime and the gas is separated completely from the liquid to form a stratified flow. Absence of the stratified flow in small rectangular channels with a small aspect ratio can be confirmed from the flow visualization work by Wambsganss et al. (1991), where a channel with an aspect ratio of 1/6 was tested.

On the other hand, Lowry and Kawaji (1988) and Wambsganss et al. (1992) have reported the effect of the flow rate on the two-phase frictional multiplier. Thus, as was suggested by Wambsganss et al. (1992), it is reasonable to take account of the effect of the mass velocity in terms of Re_{Lo} in correlation for parameter C. Consequently, C may be expressed in terms of dimensionless parameters λ, ψ and Re_{Lo} as

$$C = fn(\lambda, \psi, Re_{\mathrm{Lo}}) \tag{11}$$

$$= A\lambda^{q}\psi^{r}Re_{\mathrm{Lo}}^{s} \tag{12}$$

and constant A and exponents q, r and s may be determined through the data regression process. For the laminar–laminar flow regime, the appropriate values for the constant and the exponents are obtained from the present experiments are as in Table 4.

On the other hand, in other flow regimes (i.e., in the laminar–turbulent, turbulent–laminar and the turbulent–turbulent regimes), the exponents for λ and ψ are almost zeros, which implies that the effect of the surface tension becomes insignificant. Thus, for such cases, parameter C is merely a function of the all-liquid Reynolds number, Re_{Lo}, and appropriate values for constant A and exponent s are also listed in Table 4. In the same table, the numbers of the data points and the ranges of parameters X and Re_{Lo} covered in the present experiments are shown.

Fig. 8 compares the measured values of ϕ_{L} with the predicted values of ϕ_{L} obtained from Eqs. (5) and (12) along with the constants and exponents shown in Table 4. The predicted values of ϕ_{L} well correlates the experimental data within $\pm 10\%$. In order to assess the present correlation (Eq. (12)), the experimental results published in the works of Wambsganss et al. (1992) and Mishima et al. (1993) were compared; and, as shown in Figs. 9 and 10, Eq. (12) correlates their data points within $\pm 15\%$ and $\pm 20\%$, respectively. It should be noted that the present correlations cover wider ranges of X and Re_{Lo} compared to those of Wambsganss et al. (1992) $Re_{\mathrm{Lo}} < 2200,\ X < 1$). Their correlation well represents the high-quality case ($X < 1$), where the effect of the surface tension force may be relatively unimportant. However, for the low-quality case ($X > 1$), using the correlation by Wambsganss et al. (1992) may be meaningless since it is outside their experimental range. Moreover, another advantage of using Eq. (12) is that, unlike the correlation of Wambsganss et al. (1992), parameter C itself does not include Xas the controlling parameter. Instead, in the present work, different correlations were proposed for each flow regime. It is more desirable if parameter C can be expressed without X which is already contained in the correlation for ϕ_{L} as in Eq. (5).

4. Conclusions

In the present experimental study, a set of correlations was proposed to represent the two-phase pressure drop through horizontal rectangular channels with small heights. The correlations are

Table 4
Constant and exponents in Eq. (12) for parameter C[a]

Flow regime		A	q	R	S	Range of X	Range of Re_{Lo}	Number of data
Liquid	Gas							
Laminar	Laminar	6.833×10^{-8}	−1.317	0.719	0.557	0.776–14.176	175–1480	106
Laminar	Turbulent	6.185×10^{-2}	0	0	0.726	0.303–1.426	293–1506	52
Turbulent	Laminar	3.627	0	0	0.174	3.276–79.415	2606–17642	85
Turbulent	Turbulent	0.408	0	0	0.451	1.309–14.781	2675–17757	62

[a] Laminar: $Re_L, Re_G < 2000$; turbulent: $Re_L, Re_G > 2000$.

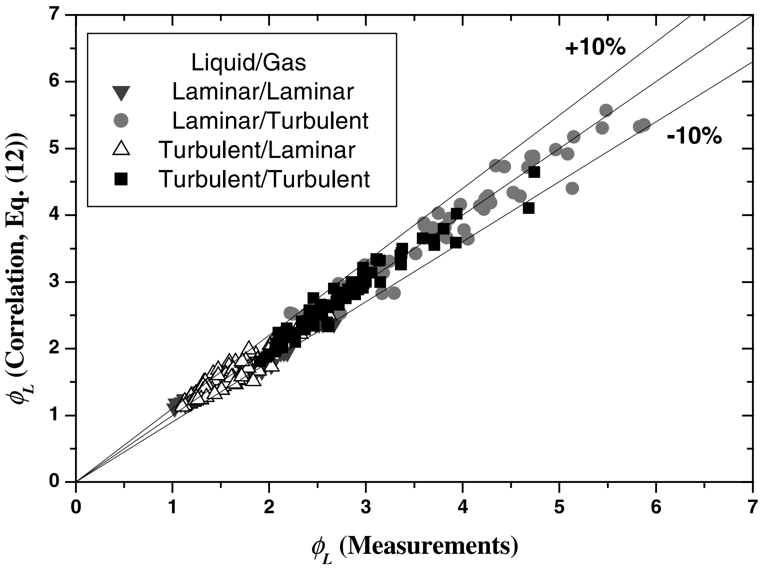

Fig. 8. Comparison between the new correlation and the present measurements.

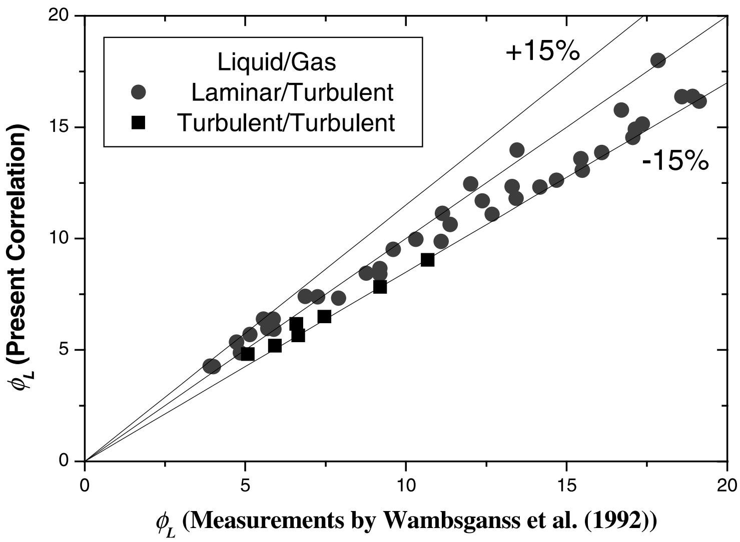

Fig. 9. Comparison between the new correlation and the measurements by Wambsganss et al. (1992) ($X < 1.0$, $Re_{Lo} < 2200$).

the Lockhart–Martinelli type but parameter C was newly defined to take account of the gap size and the flow rates of the gas and the liquid. These correlations are valid for the Martinelli parameter (X) and the all-liquid Reynolds number (Re_{Lo}) ranges of 0.303–79.4 and 175–17700, respectively. Especially, the correlations well represent the pressure drop through the extremely narrow channels in which the effect of the surface tension force is becoming of interest. As a

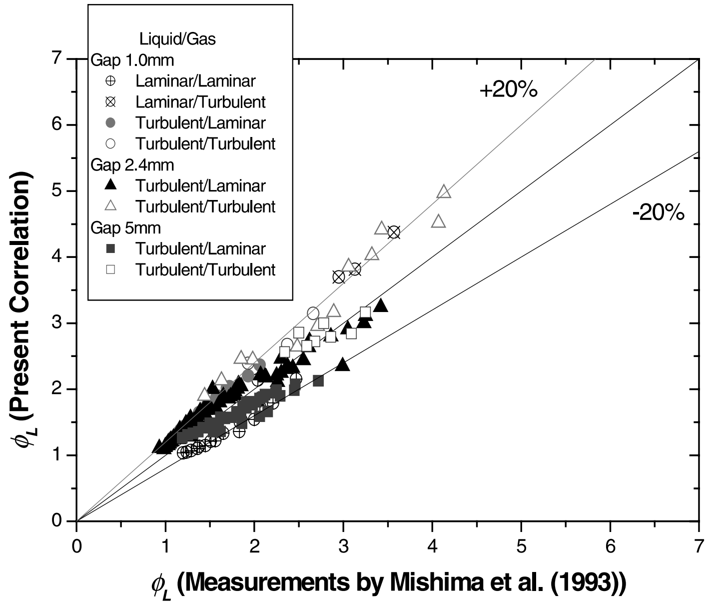

Fig. 10. Comparison between the new correlation and the measurements by Mishima et al. (1993).

whole, the correlations represent the measured data points within ±10%, and also predict the data points by Wambsganss et al. (1992) and Mishima et al. (1993) within ±15% and ±20%, respectively.

Acknowledgements

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